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Structural monitoring of the Tower of the Faculty of Engineering in Bologna using MEMS-based sensing

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ABSTRACT: Structural Health Monitoring (SHM) methodologies are taking advantage of the development of the new families of MEMS sensors and of the progress in network technologies; future systems will rely on intelligent sensors performing locally data filtering, elaboration and model identification, connected over suitable buses. This paper describes some families of multivariate models that can be used in SHM–oriented identification procedures and, in particular, the extension of AR models known as AR+noise. It describes also the implementation of a new advanced SHM system, the Teleco SHM602, in the tower of the Engineering School of Bologna University and the multivariate models identified from the data collected by this system.

KEY WORDS: SHM systems; Multivariate identification; AR models; AR+noise models.

1 INTRODUCTION

Structural Health Monitoring (SHM) methodologies [1], [2] are moving, taking advantage of the unprecedented development of sensor, microelectronics and microprocessor technologies, towards their life-long integration in new projects while still playing the role of advanced analysis tools for evaluating the state of structures not endowed with permanent monitoring systems. In particular, the introduction of advanced MEMS sensors allows the realization of systems that conjugate a contained cost with performances suitable for SHM applications.

Traditional SHM systems are essentially composed by a certain number of analog sensors (accelerometers, strain gauges, temperature sensors) connected, through signal conditioning units, to multichannel data loggers. The evaluation of the information contained in the data is then performed off-line by experts relying on suitable models of the structure to be analyzed and on the compliance of these models with the measured data [3]. The new generation of SHM systems appearing now on the market relies on designs that integrate advanced sensor technologies with distributed computational power as well as efficient implementation of identification methodologies to realize intelligent sensors that locally elaborate identification models and exchange data, information and models on a local network managed by a control/storage unit usually accessible also in a remote way.

2 MODELING DATA IN SHM APPLICATIONS

All dynamic SHM implementations rely on data, measured with a suitable sampling rate, acquired by a certain number of accelerometers mounted on the structure to be monitored. The measures obtained from these sensors are the structure response to external or internal mechanical excitations due to wind and other meteorological phenomena, vehicle traffic, seismic events, mass movements [4] or use of specific excitation hardware like mechanical shaker [5]. The state of the structure can be evaluated by comparing the responses obtained in reference (integrity) conditions with the current ones; these comparisons could be (and, sometimes, are) performed by directly extracting from the collected time series information depending only on the structure, for instance their power spectra.

In general it is preferable to avoid the storage and manipulation of the enormous amounts of data usually generated by SHM systems and to work only on some form of concentrated information extracted from the data [2], like dynamic models extracted from the data by means of identification techniques. These techniques allow not only for a very large condensation of information but can be also effectively used to separate the information contained in the acquired data sets from the observation errors due to the intrinsic noise of the sensors and to other errors due to the inevitable misfits between the considered class of models and the real process to be described. Thus the power spectrum associated with an identified model will look as (and will be) a smoothed version of that directly obtained by applying a FFT to the measured sequences that will contain many spurious lines due to additive noise.

Identified models play thus the role (common to all models) of describing the behavior of a real process to which they should be equivalent; Zadeh [6] defines system identification as *the determination on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent*. Zadeh's definition, however, must be considered only from the conceptual point of view since it cannot be strictly applied even outside the approximations implied by the identification context because mathematical models constitute always an approximation of the real systems whose actual structure and nature is always remarkably more complex. Just to mention some of the reasons of this approximation, physical systems have always a spatial distribution and cannot be exactly described by lumped parameter models; real systems are also nonlinear and, sometimes, non stationary. The most frequently used models are, however, linear, time–invariant and have finite orders; despite these limitations their behavior can constitute an excellent description of real processes over limited time intervals and in suitable ranges of input(s) variations.

Moreover, identification procedure operate in presence of process and/or observation disturbances and on limited sequences of data that make it impossible to associate a single model of the selected class of models, to an observed data set unless specific criteria are introduced. In other words, since no model will describe exactly the considered process, a specific model can be selected only by minimizing a specific cost function, associated with the planned use of the model [7]; it is thus possible to extract, from the same data, models optimized for prediction, control, filtering, fault diagnosis etc.

These considerations underline that carrying out an identification experiment requires, as first steps, the collection of suitable process data, the selection of a model class and the introduction of a cost function congruent with the planned use of the model. Model classes can concern state–space or input–output models but this aspect concerns only the structure of the algorithms for deducing the models from the data, not the quality of the obtained models. What really matters is the stochastic environment considered in the selected class of models, i.e. the description (usually by means of stochastic processes) of the errors that affect the data (process noise, additive observation errors etc.). Of course the best results are obtained by using model classes whose stochastic environment describes in the most realistic way the actual errors affecting the observations.

Another model feature that can assume great importance concerns the difference between the use of multivariate (or multivariable as these models are called in the control area) models and univariate (or scalar) ones. If we consider a process where r inputs and m outputs are present, it is possible to consider a description given by a collection of m univariate submodels where every submodel is affected by all inputs and generates a single output or a single multivariate model with r inputs and m outputs. If we disregard the overparameterization associated with the first choice (that affects, however, many important aspects like parameter uncertainty, computational loads and suitability for diagnosis applications) these solutions could be indifferently used for applications like, for instance, prediction. Only a multivariate model will, however, describe explicitly the dynamical relations between the different outputs and this makes it far superior for fault diagnosis applications.

2.1 *Multivariate AR models for SHM analysis*

In typical SHM applications the data are observations obtained by means of accelerometers properly solidarized to the structure to be monitored; the observations are performed, in a synchronous way, with a sampling frequency selected on the basis of the maximal frequency of interest. The process input, i.e. the excitation applied to the structure is only seldom measured (usually this happens only when artificial inputs are applied for test purposes by means of hammers, mechanical shakers or other methods); in almost all permanent implementations of SHM systems the excitation is given by natural phenomena, vehicle traffic, seismic events, wind pressure and is not directly measured. The available data are thus given by a sequence of L observations, $y(1), y(2), \ldots, y(L)$ where $y(t)$ denotes the vector of acceleration measures, i.e.

$$y(t) = [y_1(t)\, y_2(t)\, \ldots\, y_m(t)]^T . \tag{1}$$

A class of models frequently used to model observations of this kind is given by multivariate AR models, described by the relation

$$y(t) = Q_1 y(t-1) + Q_2 y(t-2) + \ldots + Q_\mu y(t-\mu) + e(t) \tag{2}$$

where the matrices Q_i, $(i = 1, \ldots, \mu)$ are square $(m \times m)$ coefficient matrices,

$$Q_i = \begin{bmatrix} q_{11i} & q_{12i} & \cdots & q_{1mi} \\ q_{21i} & q_{22i} & \cdots & q_{2mi} \\ \vdots & \vdots & \ldots & \vdots \\ q_{m1i} & q_{m2i} & \cdots & q_{mmi} \end{bmatrix}, \tag{3}$$

the integer μ denotes the memory of the model and

$$e(t) = [e_1(t)\, e_2(t)\, \ldots\, e_m(t)]^T \tag{4}$$

is a vector whose elements $e_i(t)$ $(i = 1, \ldots, m)$ are white processes with null expected value, $\mathrm{E}[e_i(t)] = 0$, and with variances σ_{ei}^2; these processes can be mutually correlated so that their covariance matrix is not necessarily diagonal. By denoting with z^{-1} the unitary delay operator, model (2) can be also written in the compact polynomial form

$$Q(z^{-1})\, y(t) = e(t) \tag{5}$$

where $Q(z^{-1})$ is the polynomial matrix

$$Q(z^{-1}) = I - Q_1 z^{-1} - \ldots - Q_\mu z^{-\mu}. \tag{6}$$

Model (2),(5) belongs to the family of equation error models and its optimal predictor (minimal variance and whiteness of the prediction error on every output) is given by [7]

$$\hat{y}(t) = Q_1 y(t-1) + Q_2 y(t-2) + \ldots + Q_\mu y(t-\mu) \tag{7}$$

and its prediction error $\varepsilon(t) = y(t) - \hat{y}(t) = e(t)$ coincides with the equation error. By denoting with θ a generic set of parameters of the model i.e. a generic set of entries of the matrices Q_i, the prediction error obtained by using this parameterization in predictor (7) will be denoted as $\varepsilon(t,\theta) = y(t) - \hat{y}(t,\theta)$; it coincides with $e(t)$ only when the entries of θ are the exact parameters, θ^*, of the AR process that has generated the data. Since equation error models are mainly used for prediction and control, the cost function to be minimized for estimating their parameters is given by the sum of the squares of the Euclidean norms of the prediction errors $\varepsilon(t,\theta)$

$$\begin{aligned} J(\theta) &= \frac{1}{N} \sum_{t=\mu+1}^{L} \| \varepsilon(t,\theta) \|_2^2 = \frac{1}{N} \sum_{t=\mu+1}^{L} \varepsilon(t,\theta)^T \varepsilon(t,\theta) \\ &= \frac{1}{N} \sum_{i=1}^{m} \sum_{t=\mu+1}^{L} \varepsilon_i(t,\theta_i)^2 \end{aligned} \tag{8}$$

where $N = L - \mu$ and θ_i is the vector of the coefficients appearing in the i–th row of $Q(z^{-1})$

$$\theta_i = [\, q_{i11} \dots q_{i1m} \dots q_{im1} \dots q_{imm} \,]^T . \tag{9}$$

By equating to zero the derivatives with respect to θ_i of $J(\theta)$ we obtain the Least Squares (LS) estimate of the model parameters and it can be proven that this estimate is asymptotically unbiased and, when the noise processes $e_i(t)$ are gaussian, efficient i.e. its covariance matrix equals the Kramér–Rao lower bound [7], [8]. It can be observed that the structure of cost function (8) allows separate estimates of the parameters θ_i $(i = 1, \dots, m)$; this can be performed as follows. Define the Hankel matrix of output samples

$$H(y_i) = \begin{bmatrix} y_i(1) & y_i(2) & \dots & y_i(\mu) \\ y_i(2) & y_i(3) & \dots & y_i(\mu+1) \\ \vdots & \vdots & \dots & \vdots \\ y_i(N) & y_i(N+1) & \dots & y_i(L-1) \end{bmatrix}, \tag{10}$$

the matrix

$$H = [\, H(y_1)\, H(y_2) \dots H(y_m) \,] \tag{11}$$

and the vector of output samples

$$y_i^{\circ} = [\, y_i(\mu+1)\; y_i(\mu+2) \dots y_i(L) \,]^T . \tag{12}$$

Then, under suitable excitation conditions (non singularity of $(H^T H)$), the LS estimate of θ_i is given by

$$\hat{\theta}_i = (H^T H)^{-1} H^T y_i^{\circ} \qquad (i = 1, \dots, m). \tag{13}$$

The predictions of the i–th output are the entries of the vector

$$\hat{y}_i = H\, \hat{\theta}_i = H (H^T H)^{-1} H^T y_i^{\circ} \tag{14}$$

so that the associated equation errors are given by

$$\hat{e}_i = y_i^{\circ} - \hat{y}_i = (I - H(H^T H)^{-1} H^T)\, y_i^{\circ} \tag{15}$$

and their sample covariance matrix is

$$\hat{\Sigma}_e = Y^{\circ T} (I - H(H^T H)^{-1} H^T)^2\, Y^{\circ} / N. \tag{16}$$

where

$$Y^{\circ} = [\, y_1^{\circ}\, y_2^{\circ} \dots y_m^{\circ} \,]. \tag{17}$$

The covariance matrix of the estimate of θ_i is [7]

$$\Sigma_{\theta_i} = \sigma_{ei}^2\, \mathrm{E}\left[(H^T H)^{-1} \right] \tag{18}$$

where E denotes mathematical expectation and is usually approximated as

$$\hat{\Sigma}_{\theta_i} = \sigma_{ei}^2\, (H^T H)^{-1}. \tag{19}$$

All previous steps can be easily performed on the basis of a set of observed process sequences but require a previous choice of the model memory, μ (the model order is $n = \deg \det Q(z^{-1}) = m\mu$). Of course, when the observations are generated by a true multivariate AR process, only one choice for μ and n is possible and could be estimated by applying suitable order selection criteria like FPE (Final Prediction Error), AIC (Akaike Information Criterion), MDL (Minimum Description Length) or others [7]. These criteria are usually formulated for the univariate case but can be easily extended to the multivariate one. While all previous criteria give correct results for data generated by true AR processes, it must be emphasized that this happens only in the context of computer simulations; real processes are intrinsically distributed, the correct model memory should be infinite and different criteria can lead to different evaluations.

A reliable criterion that can be applied in the identification of real processes and that can be used not only to select a proper model order but also to validate the whole identification procedure consists in checking the whiteness of the estimated equation errors $\hat{e}_i$; this happens only if the model order is sufficient and the description of the considered process by means of an AR model is acceptable. A good strategy can thus consist in starting with $\mu = 1$ and to increase μ checking, at every step, the whiteness of the sequences $\hat{e}_i$; as soon as all these sequences satisfy a proper whiteness test (for instance a χ^2 test with a number of degrees of freedom equal to 2–3 times the model memory and a reliability level of, say, 99%), a suitable model memory has been reached. It must, however, be observed that the use of higher values, while leading to overparameterized models and to higher uncertainty levels for the parameters, does not lead, usually, to a crash of the identification procedure or to worse results and this can be easily explained by the previous observation on the nature of real processes. Figure 1 shows a possible interpretation of multivariate AR models that can be seen as filters driven by the input vector $e(t)$ with transfer matrix $Q(z^{-1})^{-1}$ and output vector $y(t)$. It must be observed that the input $e(t)$ influences the output at the same time, $y(t)$, i.e. that in this system there exists an algebraical link between input and output.

It has been shown in [9] that reliable procedures for modal identification can be used to develop an efficient modal-based Structural Health Monitoring system using, for example, the AR (or ARX) coefficients as damage–sensitive parameters. When these algorithms are applied to records including the structural response to a ground motion, they can lead to unreliable results due to the fact that the hypothesis about the input (white noise) can be not fulfilled by the earthquake spectra. It is worth noting that the near–fault ground motion spectra are significantly different from those obtained in a far–field condition [10] in that usually near–fault earthquake can be viewed as an impulse; moreover intensity, ground motion spatial variations and local site conditions can influence significantly the earthquake spectra. For these reasons, in some cases the ground motion spectra can be assumed as flat at least close to the frequency range of interest. In these cases, the input of the process to be identified can be assumed as white.

The choice of the algorithms to estimate the parameters of multivariate AR models is not limited to Least Squares; another possibility concerns the use of Yule–Walker equations or of

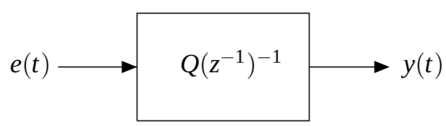

Figure 1. Interpretation of multivariate AR models

the wide class of Instrumental Variable (IV) algorithms (in fact Yule–Walker equations constitute a subcase of the IV approach where past outputs are used as instruments). These options can be used to compensate a possible lack of whiteness in the equation error sequences (by avoiding the use of the first low order equations when using Yule–Walker equations, by selecting suitable instruments when relying on IV approaches) but lead to a larger uncertainty on the parameter estimates i.e. to covariance matrices of the estimates larger than the LS one. Other possibilities concern the use of on–line algorithms, typically on–line Least Squares (weighted or not) to update a model as long as new measures are performed. The Levinson algorithm offers an elegant and efficient way to compute increasing–order AR models from data covariances. A sequence of increasing–order AR models can be estimated also directly from the data by means of Least Squares approaches.

Once that a multivariate AR model has been identified, it is also possible to obtain equivalent representations to fulfill specific needs; control applications could call, for instance, for state–space models. Other representations frequently used are the transfer matrix between the driving noise and the output, $Q(z^{-1})^{-1}$, the pulse response (AR models do not consider any measurable input; the input pulse is considered on the components of $e(t)$) and the model power spectra and cross–spectra. When the models must be used for fault diagnosis applications, as in SHM, the choice of the representation to be used can be critical. Consider, as an example, a non minimally parameterized model; its parameters could exhibit large but mutually compensated variations also in absence of significant process changes. It is thus important, to observe possible changes, to select model properties reflecting actual variations of the identified process; possible choices could concern the parameters of minimally parameterized models, model poles, frequency responses, power spectra and cross–spectra.

Another desirable feature usually absent in identified models concerns the physical significance of the models; the models obtained by means of identification techniques can be very accurate but usually lack, differently from those obtained by means of traditional modeling techniques, a direct physical meaning. This requirement and the previous one lead often to the use, in SHM applications, of the spectra and cross–spectra associated with the identified multivariate models. This information reflects well defined physical properties of the structures and can be easily linked to project–level evaluations.

Remark 1. Relation (2) is universally considered as the standard definition of multivariate AR models. This definition is, however, afflicted by severe conceptual limitations because of the implicit assumption that all channels have the same memory; thus the order of the processes described by these models can assume only values multiples of the model memory. More general and minimally parameterized representations of multivariate systems have been described in [7] and [11] and could be used also in the SHM context to obtain more physically precise descriptions of complex structures. Good results can usually be obtained also by using basic AR models like (2), (5).

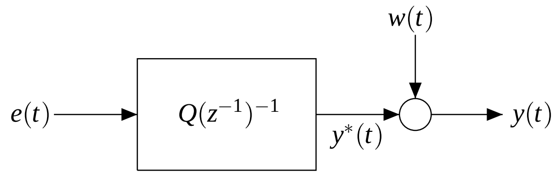

Figure 2. Structure of AR+noise models

2.2 *Advanced AR modeling: AR+noise representations*

Traditional AR models are endowed with many advantages that range from the easy estimation of their parameters by means of unbiased and efficient algorithms like LS to the stability of the associated optimal predictor (independent from the stability of the model). These models can be interpreted according to the scheme reported in Figure 1 where the equation error $e(t)$ is considered as input of a filter; in these models the equation error $e(t)$ is the only tool available to balance the different causes of misalignment between the model and the data (non linearities, process noise, observation errors, non stationarity etc.). A more sophisticated way to manage this inevitable misalignment consists in introducing a specific description of the observation errors, separating these errors from those due to other causes. AR+noise models consist in AR models whose output is considered as affected by an additive observation error (Figure 2). AR+noise models are thus described by the equations

$$y^*(t) = Q_1 y^*(t-1) + \ldots + Q_\mu y^*(t-\mu) + e(t) \quad (20)$$

$$y(t) = y^*(t) + w(t) \quad (21)$$

where

$$w(t) = [\, w_1(t)\, w_2(t)\, \ldots\, w_m(t)\,]^T \quad (22)$$

is a vector whose elements $w_i(t)$ $(i = 1,\ldots,m)$ are white processes mutually uncorrelated, uncorrelated with the entries of $e(t)$, with null expected value, $\mathrm{E}[w_i(t)] = 0$, and with variances σ^2_{wi}; the covariance matrix of $w(t)$ is thus diagonal

$$\Sigma_w = \mathrm{diag}\,\left[\,\sigma^2_{w1}\,\sigma^2_{w2}\,\ldots\,\sigma^2_{wm}\,\right]. \quad (23)$$

More general AR+noise schemes could consider additive coloured noise on the observations and/or the presence of correlations between the observation noises. The interest, in SHM implementations, of the first extension is modest while the second one, as it will be shown in the sequel, can be necessary for a realistic description of some sensors.

The identification of AR+noise models is a job more complex than the identification of AR models because it is necessary to estimate not only the parameters of $Q(z^{-1})$ and the covariance matrix Σ_e but also the covariance matrix Σ_w and, in this stochastic context, LS would lead to biased estimates.

The parameters of AR+noise models could be estimated by means of IV algorithms; the disadvantage of this solution concerns the uncertainty of the estimates and the fact that the variances of the equation errors and of the observation noise are not estimated. Another approach could be based on the mapping of the AR+noise identification problem into an EIV identification scheme, more precisely into the Frisch scheme that allows estimating, by means of a search procedure, both model parameters and the observation and process noise variances [12]. The estimate of AR+noise models by means of a Frisch–scheme approach has been described in [13] for the univariate case but can be extended to the multivariate context. An approach of this kind has the advantage of leading to a congruent solution and to be intrinsically suitable for fault diagnosis; a possible disadvantage concerns the fact that the stability of the obtained model is not assured. Another way to solve the problem could rely on the use of compensated least squares schemes, like BELS algorithms [14]. These algorithms are iterative and, usually, fast but they do not assure neither congruence nor convergence.

An approach suggested by filtering techniques applied in speech enhancement relies on the separate estimate of the variance of the additive observation noise from sequences collected in absence of signals (silent frames). This estimate is then used to compensate the presence of the observation noise reducing thus the AR+noise estimation problem to the estimation of an AR model. A procedure of this kind can be adopted also in the multivariate case and effectively applied in the SHM context. It allows also the extension to more general contexts where not all observation errors are independent and this can be of practical relevance in SHM. To illustrate this two–step procedure, consider, for an AR+noise process, the covariance matrix

$$\Sigma^* = \lim_{N\to\infty} \frac{H^{*T}H^*}{N} \tag{24}$$

where H^* has the same structure as H and is constructed with samples, $y^*(t)$, of the AR part of the model. Because of the relation $y(t) = y^*(t) + w(t)$ and of the assumption of non correlation between $e(t)$ and $w(t)$, and, consequently, between $y^*(t)$ and $w(t)$, it follows that

$$\Sigma = \lim_{N\to\infty} \frac{H^T H}{N} = \Sigma^* + \Sigma_{oe} \tag{25}$$

where Σ_{oe} denotes the covariance matrix of observation errors

$$\Sigma_{oe} = \mathrm{diag}\left[\, \sigma^2_{w1} I_\mu \, \ldots \, \sigma^2_{wm} I_\mu \,\right]. \tag{26}$$

If the covariance matrix of the observation errors, Σ_w, is known, it is possible to deduce, from (25), Σ^* and, consequently, reduce the problem to the identification of an AR process by substituting $H^T H$ with $N\Sigma$. In practical applications relation (25) will be applied to the available sample quantities by means of the relation

$$H^{*T}H^* = H^T H - N\Sigma_{oe} \tag{27}$$

and since, under the assumption of non correlation between $e(t)$ and $w(t)$, asymptotically $H^T y_i^\circ = H^{*T} y_i^{*\circ}$, the minimal–variance and asymptotically unbiased estimate of the AR model parameters is

$$\hat{\theta}_i = (H^T H - N\Sigma_{oe})^{-1} H^T y_i^\circ \qquad (i = 1,\ldots,m). \tag{28}$$

An estimate of Σ_w can be obtained by computing the sample covariance matrix of output sequences that do not contain any useful information; this can be verified by means of a whiteness test on the components of $y(t)$. The deduction of Σ_{oe} from Σ_w is then immediate.

Remark 2. The subtraction from the main diagonal of Σ of the diagonal elements of Σ_w (in blocks of μ elements) can lead to non positive definite matrices $(H^T H - N\Sigma_{oe})$ and/or to estimate unstable models. The reasons derive both from the approximation associated with the use of sample quantities and from the assumption of zero off–diagonal elements in Σ_{oe}. When this happens it is possible to modify relation (28) as follows

$$\hat{\theta}_i = (H^T H - kN\Sigma_{oe})^{-1} H^T y_i^\circ \qquad (i = 1,\ldots,m) \tag{29}$$

where $0 < k < 1$ is chosen in order to respect the condition $(H^T H - kN\Sigma_{oe}) > 0$ and the stability constraint.

3 THE SHM SYSTEM IN THE TOWER OF THE ENGINEERING SCHOOL OF BOLOGNA UNIVERSITY

The building where the tower is located has been designed by the Italian architect Giuseppe Vaccaro and was built between 1933 and 1935 (Figure 3). The tower is actually an archive capable of holding over 60,000 volumes, arranged on movable metal shelves. It is approximately 45 meters high and its structure is characterized by 4 rectangular columns which support nine concrete slabs. The measures are performed by means of a prototype of the advanced SHM system developed by

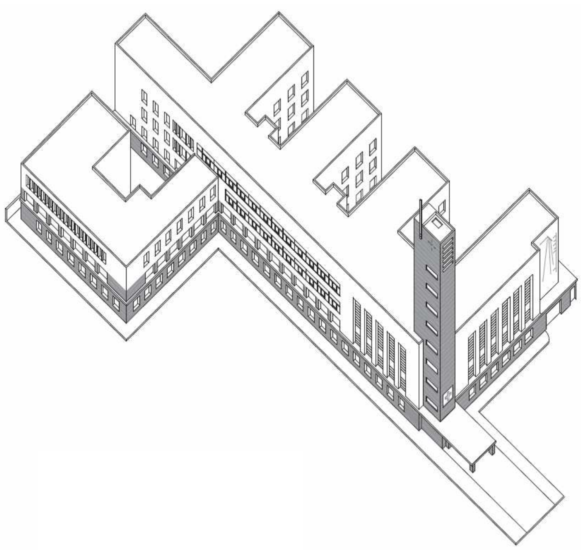

Figure 3. The Engineering School building

Figure 4. The TSM02 sensor

Teleco, the SHM602 [15], compliant with the recommendations reported in [16] and [17].

The main components of this system consist in a controller/storage unit TSD10 and in intelligent sensing units TSM02 (Figure 4) connected to the controller by means of a serial bus. Every sensing unit can send the measures of the acceleration on two orthogonal axes and that of the temperature; the sampling frequency can be selected by the user at 20 Hz, 40 Hz or 80 Hz. These sensors rely on MEMS sensors and on proprietary signal processing techniques and perform also the identification of AR+noise models [13]. The dynamic behavior of the tower is monitored by means of four TSM02 units (8 accelerometers) installed in four different floors (M1-M4 in Figures 5 and 6). Their locations have been carefully chosen in order to avoid nodal points (zero response points) on the first several vibration mode shapes. Four piezoelectric single–axis accelerometers (denoted as A1-A4 in Figures 5 and 6) have been temporarily installed in two of the previous locations for control purposes. A first set of measures has concerned the evaluation of the signal variances in absence of excitations; this can be easily performed since the building is located in a quiet area, outside traffic patterns. The results, referred to acceleration measures in *mg* (in term of variances and covariances), are reported in Table 1 where it can be observed that the noises on the x and y axes of the same unit exhibit a non negligible correlation; the covariance values associated with accelerometers of different sensors (not reported in Table 1) are, on the contrary, quite modest, more than one order of magnitude lower. This observation can be easily explained since the two accelerometers of TSM02 sensors are physically allocated on the same MEMS chip. It can also be observed that the obtained variances are perfectly aligned with the nominal values of TSM02 units with the exception of the fourth sensor whose noise level is approximately 20% lower.

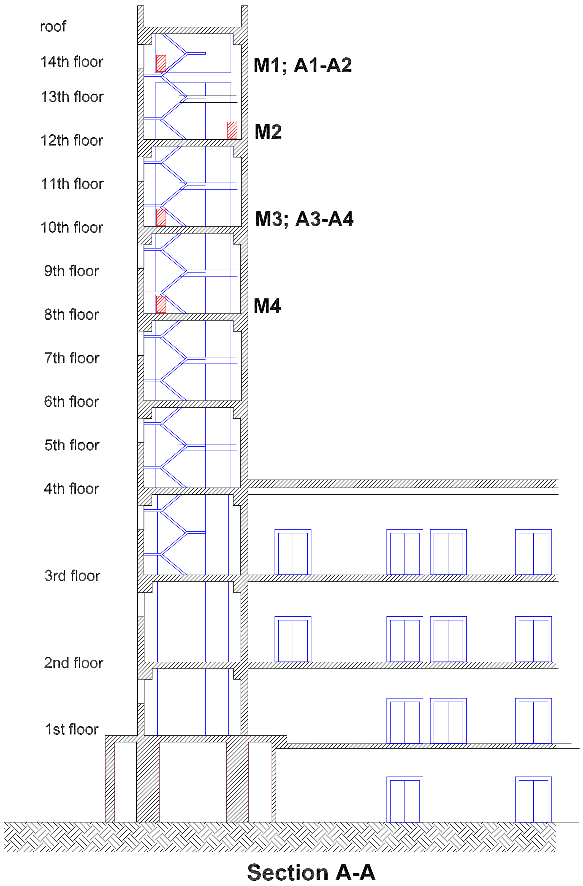

Figure 5. Accelerometer locations in the tower

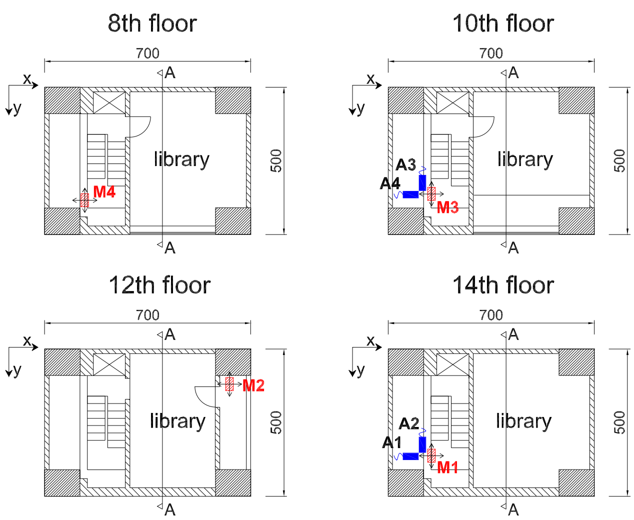

Figure 6. Accelerometer locations in the selected floors

The model actually used considered as outputs all 8 available channels and the memory selected for the model was $\mu = 10$ (order $n = 80$). The model described in the following is limited, however, to four channels in order to comply with space constraints without omitting any feature of interest of the adopted procedure. The measures considered in the construction of this reduced model are reported in Table 2. The identification has been performed by using AR+noise models and the covariance matrix of the observation noise has been constructed on the basis of the measures reported in Table 1. Since two channels of the same sensor have been inserted in the model, considering a diagonal covariance matrix Σ_w for the additive observation noise would not be congruent with the measured covariances; thus the evaluation of Σ_w that has been actually used is

$$\Sigma_w = \begin{bmatrix} 0.1087 & -0.0421 & 0 & 0 \\ -0.0421 & 0.0952 & 0 & 0 \\ 0 & 0 & 0.0986 & 0 \\ 0 & 0 & 0 & 0.0965 \end{bmatrix} \tag{30}$$

Table 1. Variances and covariances of measure noise

	σ_x^2	σ_y^2	σ_{xy}
Sensor M1	0.1087	0.0952	-0.0421
Sensor M2	0.0993	0.0986	-0.0379
Sensor M3	0.1061	0.0965	-0.0373
Sensor M4	0.0773	0.0771	-0.0315

Table 2. Model outputs

	Channel 1	Channel 2	Channel 3	Channel 4
	y_1	y_2	y_3	y_4
Sensor M1	x axis	y axis	-	-
Sensor M2	-	-	y axis	-
Sensor M3	-	-	-	y axis

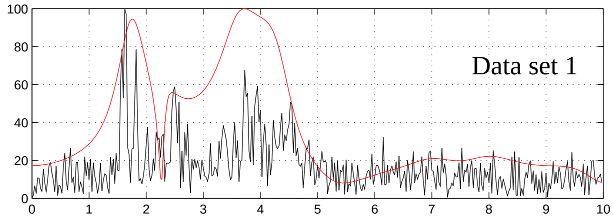

Figure 7. Power spectra of y_1; measures and model (red)

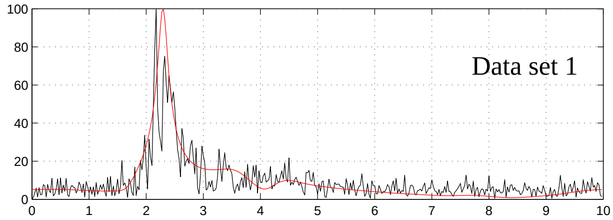

Figure 8. Power spectra of y_2; measures and model (red)

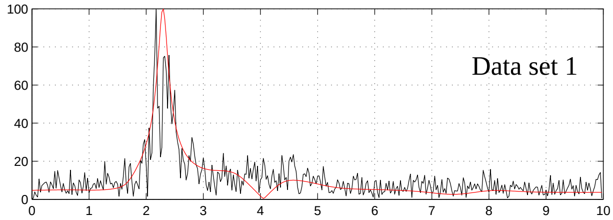

Figure 9. Power spectra of y_3; measures and model (red)

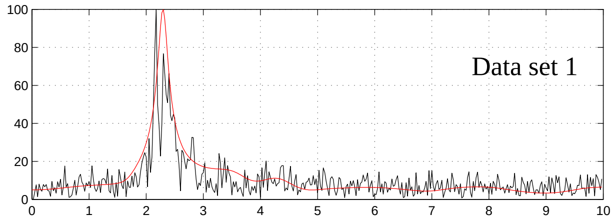

Figure 10. Power spectra of y_4; measures and model (red)

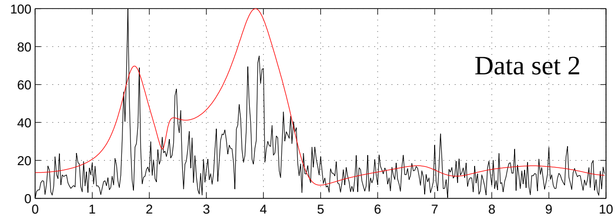

Figure 11. Power spectra of y_1; measures and model (red)

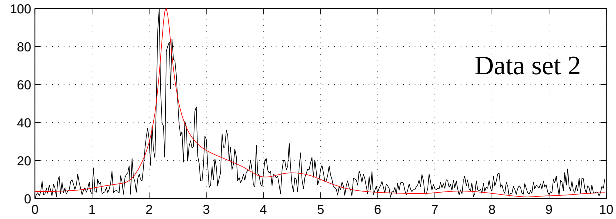

Figure 12. Power spectra of y_2; measures and model (red)

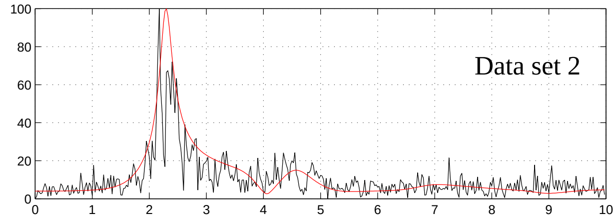

Figure 13. Power spectra of y_3; measures and model (red)

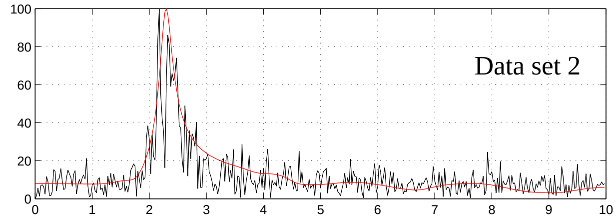

Figure 14. Power spectra of y_4; measures and model (red)

and congruent variations have been introduced in Σ_{oe} that assumes the following form

$$\Sigma_{oe} = \begin{bmatrix} 0.1087\,I_\mu & -0.0421\,I_\mu & 0 & 0 \\ -0.0421\,I_\mu & 0.0952\,I_\mu & 0 & 0 \\ 0 & 0 & 0.0986\,I_\mu & 0 \\ 0 & 0 & 0 & 0.0965\,I_\mu \end{bmatrix}. \tag{31}$$

The first data set used for the identification has been recorded on December 5, 2010 and concerns a small seismic event with magnitude 3.2 observed at a depth of 15 Km in the area of Castel San Pietro Terme, at a distance of 28 Km from Bologna.

The tests on the positive definiteness of $(H^T H - kN\Sigma_{oe})$ and on the stability of the model have shown that 0.51 was the minimum acceptable value that could be assigned to k; to leave some margin, the models have been computed assigning to k one half of the limit value. A comparison between the power spectra of the four observed sequences and those computed by means of the identified model is reported in Figures 7–10 where the scaling of the plots has been selected in order to underline the frequency peaks.

A second model has been obtained from data recorded on December 6, 2010 concerning another small seismic event with magnitude 3.0 observed at a depth of 24 Km in the same area as the previous one. The comparison between the power spectra of the four observed sequences and those computed by means of the identified model is reported in Figures 11–14. The significant peak frequencies obtained from these models are reported in Table 3. It can be observed that the main resonance frequency along the x axis is approximately 1.75 Hz while a secondary frequency is around 3.8 Hz; the resonance frequency along the y axis is approximately 2.4 Hz.

The cross–covariances between the output #2 and the outputs #1, #3 and #4 obtained with the models identified from the considered data sets are reported in Figures 15–17 that show good agreements; the worst result concerns output #1, in fact, the modes associated to the first output are those excited only marginally in the considered data sets. Similar results can be observed on remaining cross–spectra.

Finally, in Figures 18–19 a comparison between the power spectra obtained from the MEMS measures of y_1 and y_2 and

Table 3. Peak frequencies (Model 1 / Model 2)

Channel 1	1.75 / 1.75	2.45 / 2.425	3.725 / 3.85
Channel 2	2.3 / 2.3	-	-
Channel 3	2.3 / 2.3	-	-
Channel 4	2.3 / 2.3	-	-

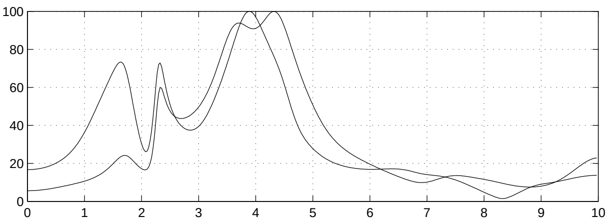

Figure 15. Cross spectra between outputs #2 and #1

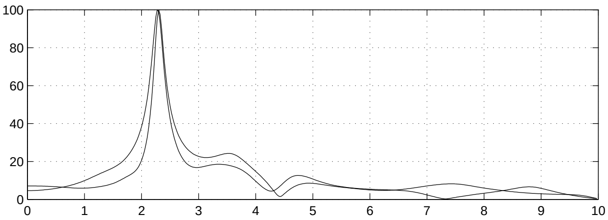

Figure 16. Cross spectra between outputs #2 and #3

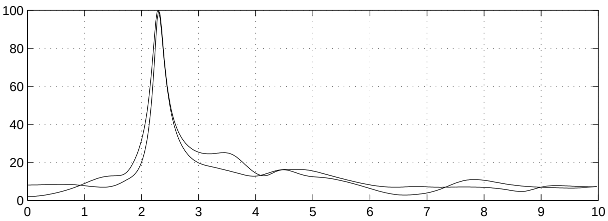

Figure 17. Cross spectra between outputs #2 and #4

those obtained with piezoelectric accelerometers placed in the same positions is shown. It can be observed that the measures are very similar in both cases. Even if the noise level of the measures is greater when MEMS-based sensing units are considered, the identified models are strongly congruent (see Figure 20). This confirms the good approximation of the real process given by the considered models and also the suitability of the SHM602 system for the performed analysis.

4 CONCLUDING REMARKS

This paper has discussed some of the problems concerning the identification of multivariate models in SHM and has outlined the potentialities offered by AR+noise models. It has also described the measures obtained from the MEMS–based SHM system Teleco SHM602 installed in the tower of the Engineering School of Bologna University and the results obtained in identifying a multivariate AR+noise model from these data.

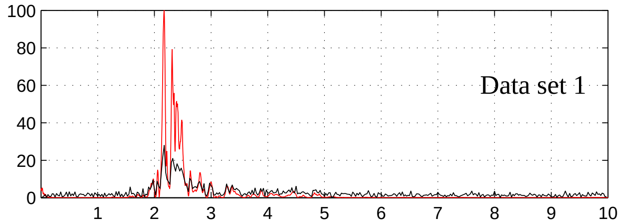

Figure 18. Power spectra of y_1: measures from MEMS-based (black) and piezoelectric accelerometers (red)

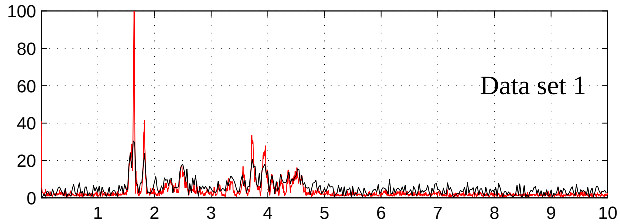

Figure 19. Power spectra of y_2: measures from MEMS-based (black) and piezoelectric accelerometers (red)

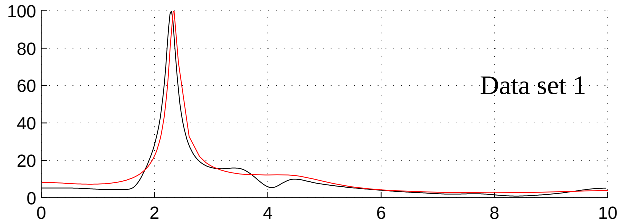

Figure 20. Power spectra of y_2: ident. models obtained from MEMS-based (black) and piezoelectric accelerometers (red)

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